V. Summary

ENSIP establishes the framework from which a contractor can evolve an engine structural development program that meets the needs of the Air Force. This is accomplished through four program elements: 1) structural design criteria, 2) structural test requirements, 3) structural data requirements, and 4) life monitoring requirements. Constituents of all four elements are contained in the latest military specification revision 5007D which has been applied to the latest Air Force turbine engine programs.

References

[1]Military Standard, "Aircraft Structural Integrity Program, Airplane Requirements," MIL-STD-1530 (USAF), Sept. 1, 1972.

[2]Military Specification, "Engines, Aircraft, Turbojet and Turbofan, General Specification for," MIL-E-5007D, Oct. 15, 1973.

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Residual Life Prediction for Surface Cracks in Complex Structural Details

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This paper reports on the development and application of an efficient technique for calculating the residual fatigue life for surface-cracked structures. The analysis treats the major three-dimensional aspects of the surface-crack problem including crack shape (planar crack with crack front curvature) and local stress variation in the structure. The surface crack is modeled with a finite number of shape-degrees of freedom in order that cyclic changes in crack shape are predicted. Necessary stress intensity factor computations are based on the use of a boundary-integral equation (BIE) model of the surface crack, together with an influence function technique for modeling the crack in an actual structure. The accuracy of the BIE crack modeling is verified with a series of buried and surface crack problems. The theoretical basis of the influence function technique is reviewed in the context of the general surface crack modeling problem. The combined residual fatigue life method is then employed to predict the life of an actual gas turbine engine disk subjected to flight-by-flight testing in a spin pit. A corner crack was initiated at the intersection of a disk bolt hole and the disk face; this crack led to the fatigue failure of the disk. The residual life analysis models this crack during the propagation phase of the disk life. Calculated residual life correlates well with the actual disk life determined experimentally.

I. Rationale

THE fracture mechanics fatigue life of engine disk structures generally may be defined so as to fall into two classes of problems: the growth of subsurface or buried cracks from intrinsic defects, or the growth of surface cracks initiated by fatigue loading of initially defect-free structural notches. The technology of stress analysis and fracture mechanics has reached the level where the fatigue life of components with buried cracks may be reliably and conservatively predicted. However, the complexity of the stress fields and crack geometry for surface-crack problems in engine structural details such as rim slots and bolt holes has precluded the use of fracture mechanics for such problems. Rather, the fatigue life of structural details with stress concentrations has been conservatively estimated by predicting the cycles to initiate a surface crack, ignoring the life which remains in the part until surface-crack growth causes failure.

Linear elastic fracture mechanics analysis forms the basis of predicting the residual fatigue life of a cracked structural element. The material is characterized in terms of its crack growth rate (da/dN) as a function of the cyclic change in the crack tip stress intensity factor (ΔK). The effects of the stress field, the crack size and shape, and the local structural geometry are enveloped by the parameter K. The primary difficulty in analyzing the growth of surface cracks is that no one value of K may be assigned to characterize the entire crack front; further, the stress state near the crack is three-dimensional due to crack front curvature and complex local geometry.

Two extreme approaches may theoretically be employed to model this three-dimensional cracking problem. An engineering approach might consist of replacing the surface crack by an "equivalent" two-dimensional, or line, crack created mathematically by combining suitable analytical models with correction functions. Unfortunately, the correction functions have been found to be unique for each problem and often can be selected only when the answer (life) is already known. The other extreme is to develop a special three-dimensional stress analysis model to reanalyze the crack geometry sequentially as it grows. This second approach requires a full three-dimensional solution for the crack at each increment in its growth history, and for each local stress distribution.

The residual life analysis procedure reported in this paper seeks to employ recent developments in fracture mechanics analysis technology to achieve the accuracy of

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three-dimensional stress analysis together with the efficiency of using "equivalent" crack models. The stress intensity factor analysis for a class of surface cracks is based on the numerical implementation of the boundary-integral equation (BIE) method of three-dimensional stress analysis. Crack growth is simulated through the use of an influence function technique which uses the stresses in the uncracked structural detail. Thus, the details of both surface crack geometry and structural shape are directly accounted for in the residual life prediction.

The next section of this paper reviews the boundary-integral equation method for fracture mechanics applications, including evaluation of the quality of the numerical results. Following the boundary-integral equation method review is a review of the influence function method including the development of a simplified crack-growth modeling procedure. The last section reports on the application of the combined tools for predicting the life of a turbine disk with a crack growing in the bolt circle. The crack growth vs flight cycles prediction is shown to correlate very well with the experimental results of a spin-pit test of the cracked disk.

II. Boundary-Integral Equation Analysis of Surface Cracks

A. Introduction

The boundary-integral equation (BIE) method of elastic fracture mechanics analysis lends itself strongly to the efficient development of a residual life prediction methodology. Surface cracks in complex structural details are inherently difficult to analyze owing to the presence of three-dimensional loading of the crack as well as the nonuniform nature of the crack tip stress intensity factor variation.

As will be described in detail, the BIE method models only the surface of the analyzed body; this contrasts strongly with finite element analyses which must model the entire body volume. Thus, for a given problem size, the BIE method is capable of significantly greater solution accuracy. Further, stress intensity factor analyses may be achieved most accurately from knowledge of the crack opening displacement field, rather than from interior stress data (e.g., see Ref. 1). The necessary crack opening displacements are directly calculated by the BIE method without the need for interior solutions. It will be shown in Sec. IIB that use of the crack opening displacements leads to excellent estimates of crack-tip stress intensity factors for three-dimensional cracks. The BIE analysis method for the elastic stress analysis of general three-dimensional problems will be briefly summarized in Sec. IIB followed by examples of the stress intensity factor analysis methodology for circular and elliptical surface cracks.

B. Review of the BIE Analysis Method

Consider the stress analysis of a three-dimensional body, as shown in Fig. 1. The boundary of the body is approximated by a set of flat triangular segments connecting M nodes. Assume that over each segment the traction vector $t_i = \sigma_{ij}n_j$,‡ and the displacement vector u_i vary linearly; nodal values of each vector are then sufficient to specify the traction and displacement vector at any point in the segment.

The basis of the BIE method is the boundary constraint integral equation

$$u_i(P)/2 + \int_s u_j(Q)T_{ij}(P, Q)dS(Q) = \int_s t_j(Q)U_{ij}(P, Q)dS(Q) \quad (1)$$

‡Subscript indices have the range (1, 2, 3); repeated indices are to be summed in the usual fashion.

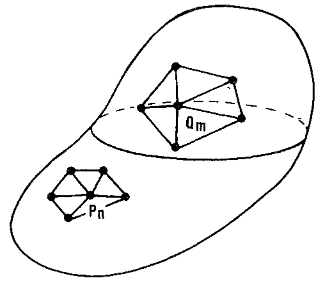

Fig. 1 Boundary segment modeling for BIE three-dimensional analysis.

which couples the displacements u_i to the tractions t_i for any points on the boundary P, Q. In Eq. (1) T_{ij} and U_{ij} are influence functions, singular at $P = Q$, which correspond to the tractions and displacements at Q due to three orthogonal point loads at P. The derivation of Eq. (1) is given in detail in Ref. 2. Making use of the modeling assumptions previously described and in Ref. 3, Eq. (1) can be reduced to a set of $3M$ algebraic equations relating nodal values of the traction vector s_i and the displacement vector, v_i.

$$[\Delta T_{ij}]\{v_i\} = [\Delta U_{ij}]\{s_i\} \quad (2)$$

In Eq. (2) the coefficient matrices are computed from closed-form expressions in terms of the elastic constants and the segment geometries.

Equation (2) becomes a square system of $3M$ equations in $3M$ unknowns for all well-posed boundary-value problems. Mixed boundary conditions are treated by swapping appropriate columns in Eq. (2) to achieve the linear expression

$$[A_{ij}]\{x_j\} = \{y_i\} \quad (3)$$

which is solved directly using standard matrix reduction algorithms. Fracture mechanics problems are analyzed by the procedure described. Stress intensity factors may be computed from the crack opening displacements or the stress field ahead of the crack as described in Refs. 3 and 4.

C. Examples of Fracture Mechanics Analysis

The verification of the BIE method for K analysis has been accomplished by a study of the buried flat elliptical crack for which analytical results are known.[5] The verification study is then used to calibrate the BIE results for elliptical surface and corner cracks. The geometry selected for study is shown in Fig. 2. A ten-inch cube containing a central elliptical crack with area, $A = \pi$ in.^{2}, (aspect ratios range from one to four) is used to simulate the infinite body. All three planes of symmetry are included in the discretized boundary model as shown in Fig. 3, with roller boundary conditions for the buried crack problems. Surface crack problems are modeled by removing rollers from a single transverse-crack plane; corner crack problems are modeled by removing the rollers from both transverse-crack planes.

The ability of the BIE method to estimate the variation of K_I along the crack front, K)s), is shown in Fig. 4.¶ The procedure for estimating $K(s)$ is to use the crack opening displacements of the nodes in the row adjacent to the crack front together with the expression

$$K_I \cong [E(2\pi)^{1/2}/4(1 - \nu^2)][v(\epsilon)/(\epsilon)^{1/2}] \quad (4)$$

In Eq. (4), v (ϵ) is the opening displacement at the normal distance ϵ from the crack front.

The BIE data in Fig. 4 have been normalized by dividing the numerical results for the buried eliptical crack by

¶Results in Fig. 4b are compared to finite element results from Ref. 6.

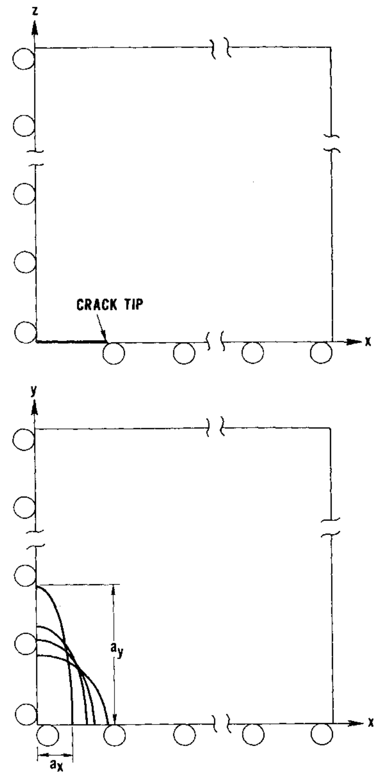

Fig. 2 Elliptical crack geometries analyzed.

the numerical results for the buried circular crack. This normalization corrects for a 7% error attributable to the discretization of the BIE model. Figure 4 shows that the BIE models are capable of excellent resolution of the relative variation of $K(s)$, particularly for the lower aspect ratio ellipses. The normalization procedure to correct for systematic modeling error is used in the life analysis calculations for the surface crack problems described.

III. Residual Lifetime Prediction Methodology

A. Introduction

Section III discusses implementation of BIE-generated $K^{\S}$ solutions for predictions of residual lifetime. Residual lifetime is defined herein as the number of constant load amplitude nominally elastic stress cycles required to grow the crack from some defined initial configuration to a final size which will produce static failure.

Paris[7] presents a currently accepted method for residual life prediction for two-dimensional cracking (i.e., a crack with constant $K(s)$ along the crack front periphery). Three-dimensional cracking has complications that are not explicitly treated by Ref. 7. The complications are the crack front variation of $K(s)$ and a tendency of crack shape, as well as size, to change during propagation.

Three concepts are introduced to treat the three-dimensional cracking complications. The first concept is a method to approximate crack geometry with a finite number of characteristic dimensions and to approximate $K(s)$ with the same number of discrete stress intensity factors, each associated with one characteristic dimension. The second concept, which is illustrated for a corner-crack model, is a particularly useful definition of the discrete stress intensity factors that facilitates the application of an influence function theory. This theory is used to build

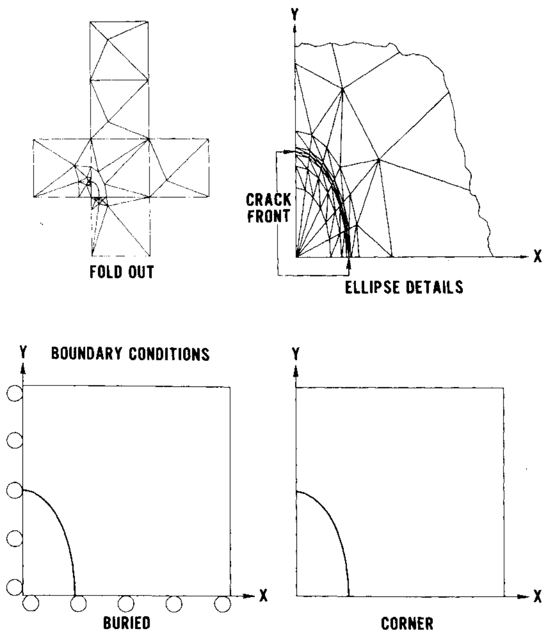

Fig. 3 BIE boundary segment model and boundary conditions for elliptical crack problems.

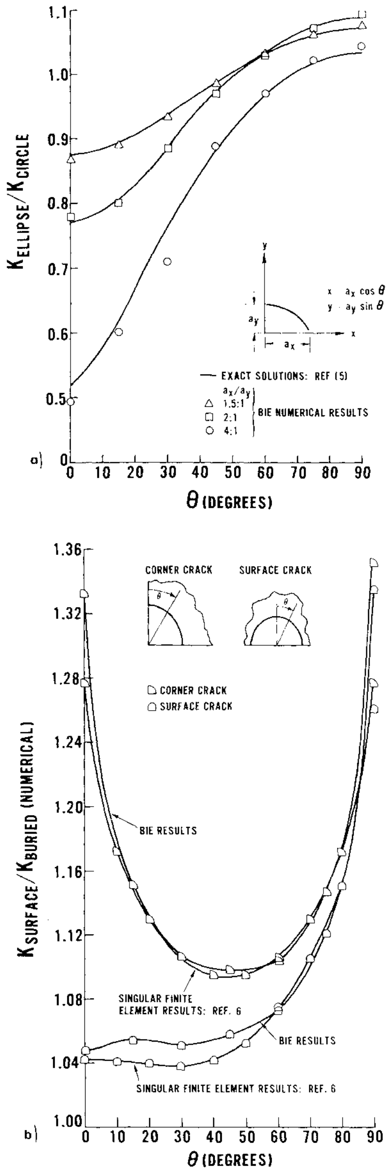

Fig. 4 $K_I(s)$ variation for a) buried elliptical cracks and for b) circular surface and corner cracks.

§The I in K_I is deleted for brevity.

an algorithm for stress intensity factor computation for general loading from crack opening displacements due to a single loading. This eliminates the need for a full three-dimensional stress analysis for each new loading or each new increment of crack growth. The third concept is application of the BIE method of stress analysis to obtain the necessary crack opening displacements as functions of geometry and position for a single loading only. This application of the BIE method, to build an efficient stress intensity factor algorithm for the elliptical corner crack problem, is explained in Sec. IIIC. The previous concepts are combined to form a procedure for residual lifetime prediction. The procedure is presented in general terms in this section and is adapted for a specific application in the next section.

B. Three-Dimensional Crack Propagation

The basis of reported life analyses is the notion of a finite number n of characteristic dimensions (a_i, $i = 1, \ldots, n$) to describe crack geometry. Crack propagation is then described by keeping track of the a_i which are named degrees of freedom or DOF. The continuous stress intensity factor function $K(s)$ is similarly approximated with a set of discrete stress intensity factors (K_i, $i = 1, \ldots, n$), each associated with an a_i. The applied general empirical model of three-dimensional propagation is then expressed by n equations

$$(da_i/dN) = F[K_i, \text{Material, Environment, History}] \quad (5)$$

where N = residual lifetime; K_i = stress intensity factor associated with a_i; and F = empirically determined function.

Each Eq. (5) states that the instantaneous cyclic growth rate da_i/dN of freedom a_i is given by the empirically determined function F. Further, Eq. (5) implies that all load and geometry information relevant to da_i/dN is contained in one and only one stress intensity factor K_i. The function F is itself independent of load and geometry and may be obtained in the traditional way[7] from simple two-dimensional laboratory specimens. The stress intensity factors K_i each contain an alternating component ΔK_i and mean value $K(mean)_i$ associated with the alternating and mean components of the stress cycle.

Residual life prediction is accomplished by formulation and solution of Eq. (5). A four-step method is employed for life prediction. The steps are: 1) Obtain F from simple specimens. F is often expressed in the form of piecewise power function of ΔK (e.g. $da/dN = C\Delta K^B$) for given K_{mean}, material, environment and history combinations. 2) Determine the uncracked structural detail's geometry, loads, and to the extent required by step 3, stress. 3) Model the propagating crack. This task includes selection of a model with an adequate number of DOF, specification of the initial and final crack configuration a_{Ii} and a_{Fi}, and definition of K_i. Further, an algorithm must be derived to compute all of the K_i as functions of stress and geometry, especially the changing crack geometry a_i. 4) Substitute K_i in Eq. (5) and solve for the life N. Steps 1 and 2 are not strongly influenced by three-dimensional cracking complications. Step 4 involves routine numerical analysis. Therefore, only step 3 is emphasized in the residual life prediction examples that follow.

C. Corner Crack in a Quarter Space (Two DOF): Model and Stress Intensity Factor Definitions

Figure 5 illustrates a model of a growing corner crack located in the x-y plane, centered at the corner/origin, and subjected to crack opening pressure $p = -\sigma_{zz}(x,y)$. The crack front is a quarter-ellipse with axes (along the coordinate axes) a_x and a_y being the two selected DOF. Having only two DOF, the model sacrifices a portion of

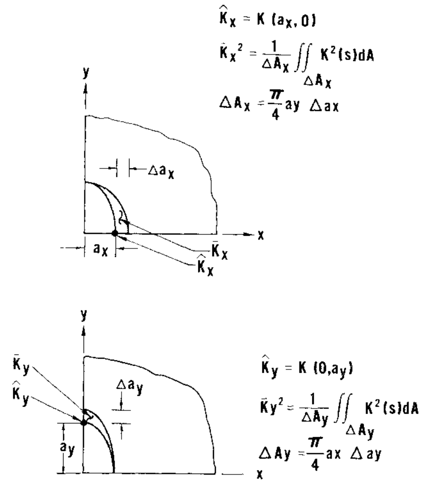

Fig. 5 Two-degree-of-freedom elliptical crack model.

the details of crack front geometry and $K(s)$ variation inherent in more complex models having more than two DOF. However, the two-DOF model is considered a worthwhile approximation of three-dimensional cracking since it does permit some change of crack shape during propagation and promises relative ease of application. The choice of a quarter-ellipse model is especially expedient since published solutions for the buried elliptical crack are heavily utilized herein to help compute accurate stress intensity factors for the corner crack. Having chosen a model, the stress intensity factors K_i (K_x and K_y) are to be defined and computed. Since the question of proper K_i definition for residual life computation may be debated,[8] two alternatives have been considered. The alternatives are illustrated in Fig. 5 as local (or end-point values) $\hat{K}_x$ and $\hat{K}_y$, and "local average" values $\bar{K}_x$ and $\bar{K}_y$ as described following.

The $\bar{K}_i$, rather than the $\hat{K}_i$, definition is chosen in Ref. 8 and herein for residual life prediction for two reasons: 1) relative ease of the $\bar{K}_i$ computation (given a tool like the BIE method) under general loads through use of an influence function theory; and, 2) previous successful experience with this definition in the residual life estimates of complex structures. A typical application is presented in a later section.

Each $\bar{K}_i$ is related to the strain energy release rate obtained by pertubation of its corresponding freedom a_i alone. Thus,

$$\bar{K}_x \equiv (HG_x)^{1/2} \quad (6)$$

$$\bar{K}_y \equiv (HG_y)^{1/2} \quad (7)$$

where H is an appropriate elastic modulus equal to $E/(1 - \nu^2)$ for an isotropic** material in plane strain. The G_i are strain energy release rates associated with creation of the shaded surface areas ΔA_i in Fig. 5

$$G_x = (\partial V/\partial A)_{a_y} = (4/\pi a_y)(\partial V/\partial a_x) \quad (8)$$

$$G_y = (\partial V/\partial A)_{a_x} = (4/\pi a_x)(\partial V/\partial a_y) \quad (9)$$

where the crack face area is given by $A = \pi a_x a_y/4$. As indicated by Fig. 5 and proven in Ref. 4, $\bar{K}_i$ also represents the root-mean-square value of $K(s)$ in the shaded areas ΔA_i exposed by pertubation of a_i alone.

D. Influence Function Theory for Stress Intensity Factor Computation

Reference 8 modifies an influence function theory due to Rice[9] to obtain exact solutions for $\bar{K}_x$ and $\bar{K}_y$ for a two-

**Reference 9 indicates that H may also be expressed in terms of general anisotropic material compliances.

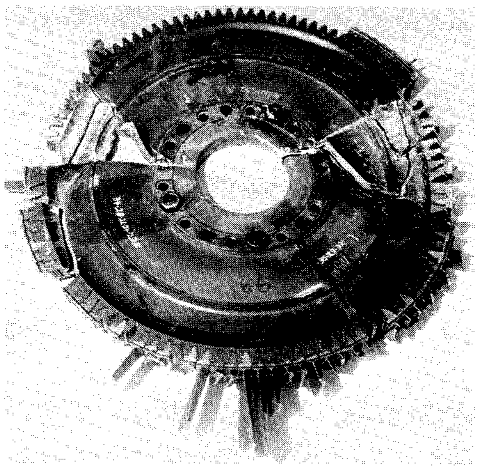

Fig. 6 Turbine disk failed in spin-pit fatigue test.

DOF buried elliptical crack under arbitrary Mode I loads. This paper, taking full advantage of the buried ellipse solutions, applies the BIE method to compute crack opening displacements, influence functions, and $\bar{K}_x$ and $\bar{K}_y$ for the corner crack under arbitrary load.

The $\bar{K}_i$ may be computed with influence function theory[8] from

$$\bar{K}_x = \int\int_A h_x(x, y, a_x, a_y)\sigma_{zz}(x, y)dA \qquad (10)$$

$$\bar{K}_y = \int\int_A h_y(x, y, a_x, a_y)\sigma_{zz}(x, y)dA \qquad (11)$$

where σ_{zz}, called the uncracked stress, is the normal stress in the uncracked solid at the crack locus. The influence functions h_x and h_y are computed from knowledge of the crack opening displacement $w = u_z$ as a function of position and geometry (a_x and a_y in the case of the corner crack model) for a single arbitrary uncracked stress, called a "reference stress." Reference 8 derives the key equations for the influence function theory as

$$h_x = \left[\frac{\partial(\int\int_A \sigma_{zz}^* w^* dA)}{H\partial A}\right]^{-1/2}_{a_y} \left(\frac{\partial w^*}{\partial A}\right)_{a_y} \qquad (12)$$

$$h_y = \left[\frac{\partial(\int\int_A \sigma_{zz}^* w^* dA)}{H\partial A}\right]^{-1/2}_{a_x} \left(\frac{\partial w^*}{\partial A}\right)_{a_x} \qquad (13)$$

where the superscript* denotes a single reference stress $\sigma^*_{zz}(x,y)$ whose choice, in theory, does not affect h_x and h_y which depend on position and geometry alone. Therefore, accurate computation of h_x and h_y for any convenient σ^*_{zz} eliminates any future need to include the corner crack in stress analysis models.

The BIE method is used to obtain the necessary crack opening displacement data for a uniform reference stress $\sigma^*_{zz}(x,y) = \sigma_o$ for the aforementioned models (Fig. 3) of the buried ellipse and the corner quarter-ellipse. Data

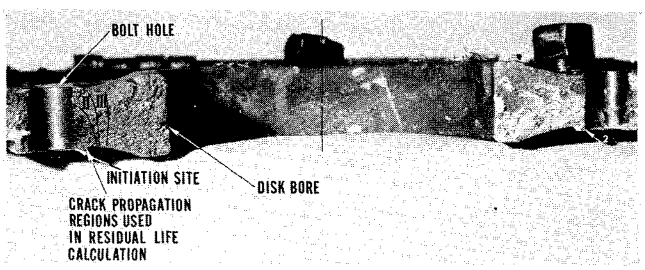

Fig. 7 Cross section showing primary failure origin and crack growth regions.

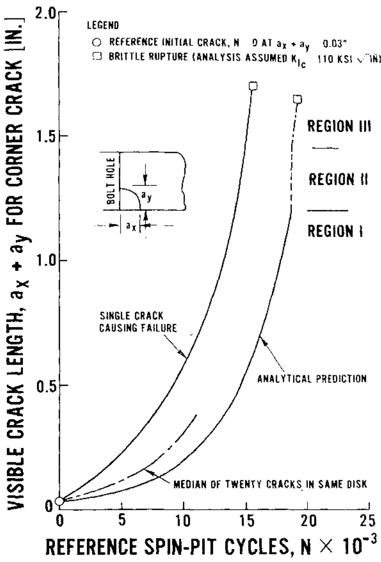

Fig. 8 Comparison of predicted and actual crack length vs spin-pit cycles.

generation was accomplished with 12 full three-dimensional stress analysis computer runs requiring a total of 22 minutes of C.P.U. time on the IBM 370-168 computer. Corner crack displacement data are expressed as a mathematical function w^* (x, y, a_x, a_y) by means of standard least-square data-fitting techniques. The function is substituted into Eq. (12) and (13) to compute influence functions h_x (x, y, a_x, a_y) and h_y (x, y, a_x, a_y) for storage in a computer algorithm. The algorithm, which is nothing more than a numerical integration scheme to evaluate Eq. (10) and (11), computes stress intensity factors for the growing corner crack subjected to any specified uncracked stress σ_{zz} (x,y). Section IV presents a typical application of the algorithm to obtain an estimate for residual lifetime in a fairly complex structural detail.

IV. Residual Life Prediction for a Structural Detail

A. Structure and Fatigue Experiment Description

Figures 6 and 7 show the cracked structural detail analyzed herein as part of the development and corroboration of residual life prediction methodology. The structure is a low-pressure turbine rotor disk removed from engine operation with cracks emanating from the corners of all twenty bolt holes. The material is Incoloy 901, a nickel-steel alloy commonly used for disk forgings. As illustrated in Fig. 7, each crack proceeds along the bolt hole surface through the thickness (freedom a_y) as well as radially inward toward the disk bore (freedom a_x).

A high-temperature cyclic spin-pit experiment was performed to ascertain the structure's residual lifetime to a condition of brittle failure. The spin-pit experiment was designed to simulate, as closely as possibly, the takeoff condition temperature field and the stress cycle field encountered in the engine operating at ground idle, going to takeoff power, then back to ground idle. Figure 8 shows the results of the experiment in terms of total visible crack length ($a_x + a_y$ for the corner crack) as a function of spin-pit cycles N for the single crack that caused failure and the crack exhibiting median growth rate of the 20 bolt-holes. Unfortunately, only the total visible crack length was reported.

B. Residual Life Analysis

The residual life methodology described previously is applied to the bolt-hole problem. Salient features of the four-step procedure are outlined. The first step is procurement of appropriate Incoloy 901 material crack propagation data from simple laboratory specimen testing. Center notched sheets machined in the proper orientation from disk forgings had been previously tested at several combinations of temperature and mean stress. The specimen fatigue crack progression duplicated that of the spin test in all important qualitative details. Net section stresses were nominally elastic, shear lips were relatively small and cracks stayed in their respective Mode I symmetry planes for specimens and disk. In addition, specimen geometry and stress amplitude variations were imposed. This study verified that the crack propagation rate could be adequately correlated with stress intensity factor for the material/load/environment combinations tested.

Standard crack propagation data reduction (Ref. 7) yields the median data

$$(da/dN) = 1.3 \times 10^{-10}\Delta K^{3.1}[\text{in./cycle}], 12 < \Delta K < 70\ k\text{si(in.)}^{1/2} \quad (14)$$

for temperature (500°F) and mean K ($K_{\text{mean}} = 0.60\ \Delta K$) at the bolt hole. As discussed previously, Eq. (14) is interpreted as

$$da_x/dN = 1.3 \times 10^{-10}\Delta \bar{K}_x^{3.1} \quad (15)$$

$$da_y/dN = 1.3 \times 10^{-10}\Delta \bar{K}_y^{3.1} \quad (16)$$

with respect to the corner crack algorithm represented by Eq. (10) and Eq. (11).

The second step is stress analysis of the uncracked structure. Since the crack need not be included in the stress analysis, standard in-house programs are used for this step. Table 1 shows the complex nature and high gradients of the uncracked stress with point estimates of the transverse stress concentration factor.

The third step is modeling the propagating crack. Figure 7 defines three regions of crack propagation chosen for separate modeling. Region I defines crack propagation from a measured initial configuration ($a_x + a_y = 0.03$ in.) to a configuration three-fourths ($a_y = 0.64$ in.) of the distance through the 0.85-inch thickness. Region II defines the transition to a one-DOF, through-thickness crack while Region III defines the growth of the through-thickness crack to a critical dimension causing brittle failure.

The model chosen for Region I is the most important part of the residual lifetime analysis predicated on the fact that, barring dramatic crack arrestment features, a small crack with its lower ΔK grows slowly compared to a large crack. Therefore, the majority of residual lifetime occurs within Region I. The two-DOF corner crack model (Fig. 5) is chosen for estimation of the Region I portion of residual life. Computation of $\bar{K}_x$ and $\bar{K}_y$ is accomplished by substitution of the stress data (Table 1) and crack dimensions (a_x, a_y) into a small computer subroutine that stores the BIE-generated influence functions and numerically evaluates the integrals in Eqs. (10) and (11). Initial values of a_x and a_y are not explicitly known, although their initial sum is 0.03 in. A procedure for initial value estimates, compatible with the applied model, is used to calculate the initial a_x and a_y values. The final Region I configuration is, as previously stated, given by $a_y = 0.64$ in.

Rigorous K computation for cracks in Region II is beyond the scope of the present investigation. Fortunately, the fraction of total structural lifetime associated with Region II is small in this instance and is ignored in the analysis. The Region III crack growth lifetime is also small and is estimated with two-dimensional K-formulas and the accepted methods of Ref. 7.

Table 1 Transverse stress concentrations near the bolt hole ($\sigma_{zz}(x,y)/\sigma_{\text{nominal}}$)

Distance from bolt-hole surface (in.)	$\sigma_{zz}(x,y)/\sigma_{\text{nominal}}$ Disk front face	Disk back face
0.	2.20	1.96
0.0058	2.11	1.89
0.0115	2.05	1.83
0.0211	1.95	1.74
0.0307	1.84	1.67
0.0519	1.70	1.52
0.0709	1.56	1.40
0.105	1.45	1.28
0.137	1.34	1.20
0.212	1.24	1.11
0.287	1.18	1.05
0.437	1.15	1.03
0.587	1.18	1.05
1.037	1.34	1.20

[a] Crack origin at the intersection of the bolt hole and the disk front face.
[b] σ_{zz} (x,y) distribution in crack plane based on linear interpolation of this data in the (x,y) directions

The fourth and final step for Region I life estimates is substitution of previously described estimates of $\bar{K}_x$, $\bar{K}_y$, and initial and final values of a_x and a_y into Eq. (15) and (16) for solution for the lifetime N. Studies have shown that equation sets like Eq. (15) and (16) are usually well-behaved and may be easily solved with any standard numerical technique. A form of Hamming's Predictor-Corrector Method is employed in a computer program which also contains the previously mentioned stress intensity factor algorithm subroutine. Program data input coding and run costs are usually less than five dollars for each residual lifetime computation.

Figure 8 presents the lifetime analysis results by comparing measured and computed crack length ($a_x + a_y$) as functions of the number of spin-pit cycles. Since the deterministic analysis herein used median laboratory crack-propagation data, comparison between computed and measured median crack propagation is judged to be most meaningful. Correlation between the two median-based curves is very good with most discrepancy occurring at the smallest crack lengths. Among the many possible causes for this discrepancy, it is speculated that the possible plastic history of material closest to the notch is most important. The good correlation between measured and computed lifetimes shown in Fig. 8 has usually been achieved for other applications with accurately known material crack propagation and cyclic stress data.

Summary and Conclusions

An efficient, accurate technique for predicting the fatigue life of surface cracks in areas of concentrated stresses has been reported. The technique is based on the use of the boundary-integral equation (BIE) method for the elastic stress analysis, and an influence function method for modeling key surface crack growth parameters. The BIE method is shown to be capable of accurately modeling the three-dimensional characteristics of surface cracks, including the crack front stress intensity factor distribution. The BIE model also provides numerical data for the dependence of crack opening displacements on crack dimensions that are used by the influence function method.

The paper shows that a surface crack may be modeled, for purposes of the current study, by a two-parameter, or degree of freedom (DOF), crack shape. The crack shape is taken to be an ellipse with variable length major and minor axes. A set of two crack-tip stress intensity factors appropriate to the two-DOF crack growth model is derived for crack growth modeling. The paper indicates how more complex crack shapes may be modeled by the same procedure through the use of additional DOF and reference crack shapes.

The influence function method is based on the characterization of the crack opening displacements for various values of the two DOF, for a simple reference stress loading of the cracked geometry. These numerical results are then used to analytically derive values of the two effective stress intensity factors for an arbitrary uncracked stress state such as due to the presence of a local notch. The crack shape and size is then computed as a function of load cycles by the use of the usual linear elastic fracture mechanics modeling of crack growth.

The residual fatigue life prediction technique is illustrated for the significant problem of a corner crack growing from a bolt hole in an engine disk. The study shows that the technique is capable of modeling the majority of the crack growth process, for which the simple elliptical crack shape model is applicable. Correlation of the numerical results with an in-house engine disk test is reasonably good and demonstrates the ease of use of the technique that is necessary for general application.

Based on the crack modeling presented in this paper, a corner crack occurring in any structural detail of large dimensions compared to the initial crack may be modeled by using the uncracked stress state. Extension of the technique to other crack shapes and structural geometries is direct. Extension requires only the specification of a crack model with an appropriate number of DOF together with an appropriate BIE characterization of the crack and structural geometry. Thus, the majority of surface crack problems are brought within the scope of an efficient residual fatigue life prediction technique.

References

[1]Cruse, T. A., "Numerical Evaluation of Elastic Stress Intensity Factors by the Boundary-Integral Equation Method," *The Surface Crack: Physical Problems and Computational Solutions,* edited by J. L. Swedlow, American Society of Mechanical Engineers, New York, 1972, pp. 153–170.

[2]Cruse, T. A., "Numerical Solutions in Three Dimensional Elastostatics," *International Journal of Solids and Structures,* Vol. 5, 1969, pp. 1259–1274.

[3]Cruse, T. A., "An Improved Boundary-Integral Equation Method for Three Dimensional Elastic Stress Analysis," *Computers and Structures,* Vol. 4, 1974, pp. 741–754.

[4]Chan, S. K., Tuba, I. S., and Wilson, W. K., "On the Finite Element Method in Linear Fracture Mechanics," *Engineering Fracture Mechanics,* Vol. 2, No. 1, 1970, pp. 1–18.

[5]Green, A. E. and Sneddon, I. N., "The Distribution of Stress in the Neighborhood of a Flat Elliptical Crack in an Elastic Solid," *Proceedings of the Cambridge Philosophical Society,* Vol. 46, 1950, pp. 159–163.

[6]Tracey, D. M., "Finite Elements for Determination of Crack Tip Elastic Stress Intensity Factors," *Engineering Fracture Mechanics,* Vol. 6, 1971, pp. 255–265.

[7]Paris, P. C., Gomez, M. P., and Anderson, W. E., "A Rational Analytic Theory of Fatigue," *The Trend in Engineering,* Vol. 13, University of Washington, Seattle, pp. 9–14.

[8]Besuner, P. M., "Residual Life Estimates for Structures with Partial Thickness Cracks," to be presented at the ASTM Eighth National Symposium on Fracture Mechanics, Providence, R.I., American Society for Testing and Materials, 1974.

[9]Rice, J. R., "Some Remarks on Elastic Crack-Tip Stress Fields," *International Journal of Solids and Structures,* Vol. 8, 1972, pp. 751–758.